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$$\therefore S = \frac{2 \int_0^a \int_0^{2r} \int_0^{\sqrt{(2rx-x^2)}} \frac{adrxdy}{\sqrt{(2ax-x^2-y^2)}}}{\int_0^a dr} = \pi \int_0^a \int_0^{2r} drdx = \pi a^2,$$

the area of a great circle of the sphere.

If  $y' = \sqrt{(2ax-x^2)}$ ,  $z = \sqrt{(2ax-x^2-y^2)}$ , then we have

$$\begin{aligned} V &= \frac{4}{a} \int_0^a \int_0^{2r} \int_0^{y'} \int_0^z drdxdydz = \frac{4}{a} \int_0^a \int_0^{2r} \int_1^{y'} (2ax-x^2-y^2) drdxdy \\ &= \frac{2}{a} \int_0^a \int_0^{2r} \left[ (2ax-x^2) \sin^{-1} \left( \frac{2r-x}{2a-x} \right) + x \sqrt{[2(a-r)(2r-x)]} \right] drdx \\ &= \frac{8}{9a} \int_0^a \left[ 3a^3 \tan^{-1} \left( \frac{r}{a-r} \right) - (3a^2 + 2ar - 8r^2) \sqrt{(ar-r^2)} \right] dr = \frac{8}{9} a^3 \pi \end{aligned}$$

$= \frac{8}{9}$  of the volume of the sphere.

Also solved by *G. B. M. ZERR*, who gets  $\frac{1}{2}\pi R^3$  as a result for the second part of the problem.

A partial solution was received from *F. P. Matz*.

Professor Walker should have received credit in the last issue for solution of problem 108.

• 110. Proposed by *G. B. M. ZERR*, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Find the average area of the triangle formed by joining three random points taken on the surface of a regular hexagon, two on one side of a diagonal and the third on the other side.

Solution by the PROPOSER.

Let  $ABCDEF$  be the hexagon;  $P, R$  the random points above the diagonal  $AD$ ;  $Q$  the random point below the diagonal. Through  $P, R, Q$  draw  $LL', MM', NN'$  parallel to  $AD$ , and  $TT'$  perpendicular to  $AD$  through  $O$ . It is only necessary to consider the relative positions in which the line  $MM'$  lies between  $LL'$  and  $NN'$ .

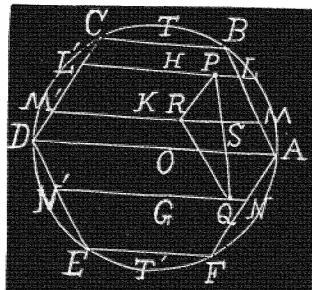
Let  $CB=OA=a$  be the side of the hexagon,  $OH=u$ ,  $OG=v$ ,  $OK=w$ ,  $HP=x$ ,  $GQ=y$ ,  $KR=z$ ,  $KS=t$ ,  $HL=x'$ ,  $GN=y'$ ,  $KM=z'$ .

Then  $OT = \frac{1}{2}a\sqrt{3} = u'$ ,  $x = (a\sqrt{3} - u)/\sqrt{3}$ ,  
 $y' = (a\sqrt{3} - v)/\sqrt{3}$ ,  $z' = (a\sqrt{3} - w)/\sqrt{3}$ ,  
 $t = y - [(y-x)(v+w)]/(u+v)$ .

Area  $PQR = \frac{1}{2}(t-z)(u+v) = A$ , when  $t > z$ .

Area  $PQR = \frac{1}{2}(z-t)(u+v) = A$ , when  $t < z$ .

The limits of  $u$  are 0 and  $\frac{1}{2}a\sqrt{3}$ ; of  $v$ , 0 and  $\frac{1}{2}a\sqrt{3}$ ; of  $w$ , 0 and  $u$ ; of  $x$ ,  $-x'$  and  $x'$ ; of  $y$ ,  $-y'$  and  $y'$ ; of  $z$ ,  $-z'$  and  $t$ , and  $t$  and  $z'$ .



The whole number of ways the three points can be taken is  $\frac{81\sqrt{3}}{64}a^6$ .

Doubling, since the halves are interchangeable, we get for average area of triangle :

$$\begin{aligned}
 \Delta &= \frac{128}{81\sqrt{3}a^6} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^u \int_{-x'}^{x'} \int_{-y'}^{y'} \left[ \int_{-z'}^t A dz + \int_t^{z'} A_1 dz \right] dudvdwdxdy \\
 &= \frac{64}{81\sqrt{3}a^6} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^u \int_{-x'}^{x'} \int_{-y'}^{y'} \left[ \frac{1}{3}(a\sqrt{3}-w)^2 + \left( y - \frac{(y-x)(v+w)}{u+v} \right)^2 \right] \\
 &\quad \times (u+v) dudvdwdxdy \\
 &= \left(\frac{2}{3}\right)^7 \frac{1}{a^6} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^u \int_{-x'}^{x'} \left[ 3(a\sqrt{3}-w)^2 (a\sqrt{3}-v)(u+v) \right. \\
 &\quad \left. + \frac{(a\sqrt{3}-v)^3 (u-w)^2 + 9x^2 (a\sqrt{3}-v)(v+w)^2}{u+v} \right] dudvdwdx \\
 &= \left(\frac{2}{3}\right)^8 \frac{\sqrt{3}}{a^6} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^u \left[ 3(a\sqrt{3}-u)(a\sqrt{3}-v)(a\sqrt{3}-w)^2 (u+v) \right. \\
 &\quad \left. + \frac{(a\sqrt{3}-u)(a\sqrt{3}-v)^3 (u-w)^2 + (a\sqrt{3}-u)^3 (a\sqrt{3}-v)(v+w)^2}{u+v} \right] dudvdw \\
 &= \left(\frac{2}{3}\right)^8 \frac{1}{\sqrt{3}a^6} \int_0^{\frac{1}{2}(a\sqrt{3})} \int_0^{\frac{1}{2}(a\sqrt{3})} \left[ 9a^3 \sqrt{3} (a\sqrt{3}-u)(a\sqrt{3}-v)(u+v) \right. \\
 &\quad \left. - 3(a\sqrt{3}-u)^4 (a\sqrt{3}-v)(u+v) + (a\sqrt{3}-u)^3 (a\sqrt{3}-v)(u+v)^2 \right. \\
 &\quad \left. + \frac{u^3 (a\sqrt{3}-u)(a\sqrt{3}-v)^3 - v^3 (a\sqrt{3}-u)^3 (a\sqrt{3}-v)}{u+v} \right] dudv \\
 &= \left(\frac{2}{3}\right)^5 \frac{1}{27\sqrt{3}a^6} \int_0^{\frac{1}{2}(a\sqrt{3})} \left[ 8\sqrt{3}au^6 - 42a^2u^5 + 127\sqrt{3}a^3u^4 - 480a^4u^3 + 81\sqrt{3}a^5u^2 \right. \\
 &\quad \left. + 216a^6u + 16u^3(9a^4 - u^4) \log\left(\frac{2u+a\sqrt{3}}{2u}\right) \right] du = \frac{1507\sqrt{3}}{11664}a^2.
 \end{aligned}$$

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#### MISCELLANEOUS.

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104. Proposed by HARRY S. VANDIVER, Bala, Pa.

A Theorem of Fermat. The area of a right angled triangle with commensurable sides cannot be a square number. [Cf. Chrystal's *Algebra*, Vol. II., page 535.]